Faraday Rotation as a Diagnostic of Galactic Foreground Contamination of CMB Maps

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2 February 2008

ABSTRACT
We present a diagnostic test of possible Galactic contamination of cosmic microwave background sky maps designed to provide an independent check on the methods used to compile these maps. The method involves a non-parametric measurement of cross-correlation between the Faraday rotation measure (RM) of extragalactic sources and the measured microwave signal at the same angular position. We argue that statistical properties of the observed distribution of rotation measures are consistent with a Galactic origin, an argument reinforced by a direct measurement of cross-correlation between dust, free–free and synchrotron foreground maps and RM values with the strongest correlation being for dust and free–free. We do not find any statistically compelling evidence for correlations between the RM values and the COBE DMR maps at any frequency, so there is no evidence of residual contamination in these CMB maps. On the other hand, there is a statistically significant correlation of RM with the preliminary WMAP individual frequency maps which remains significant in the Tegmark et al. Wiener-filtered map but not in the Internal Linear Combination map produced by the WMAP team.

Key words: cosmic microwave background – Cosmology: observations – methods: statistical

1   INTRODUCTION

The DMR (Differential Microwave Radiometer) instrument on board the COBE satellite ushered in the era or precision cosmology by measuring primordial temperature fluctuations of order $\Delta T/T = 10^{-5}$ in the cosmic microwave background (CMB) radiation (Smoot et al. 1992). The WMAP (Wilkinson Microwave Anisotropy Probe) recently took CMB cosmology onto a higher level by mapping the last scattering surface with much higher angular definition and greater sensitivity (Bennett et al. 2003; Hinshaw et al. 2003). But before either of these experiments could produce cosmological information various sources of contaminating radiation had to be eliminated.

Among the foregrounds that may affect CMB observations are Galactic dust, synchrotron and free–free emission, extragalactic point sources and the Sunyaev-Zel'dovich effect due to hot gases in galaxy clusters. These fluctuations were measured by the COBE-DMR instrument on angular scales larger than 7° and at 3 different frequencies (31.5, 53 and 90 GHz) corresponding to a window where the Galactic emissions are minimum. Instruments such as COBE-DMR, which probe large angular scales, are most sensitive to the diffuse foreground emission of our galaxy (Gawiser & Silk

2000). Synchrotron emission associated with the motion of electrons in the Galactic magnetic field dominates at $\nu \lesssim$ 20 GHz. Free-free emission, due to electron-ion collisions in the interstellar medium, dominates in the range 25-70 GHz. Galactic dust grains absorb UV and optical light from the interstellar radiation field and emit the energy in the far-infrared. This foreground dominates at $\nu \gtrsim$ 90 GHz. CMB instruments can identify foregrounds by their spectral signatures across a range of frequencies, making reliable foreground subtraction feasible. WMAP was designed to make accurate measurements of the CMB down to angular scales of ∼ 12 arcmin; to allow efficient foreground subtraction this experiment mapped the full sky at five widely separated frequencies from 23 GHz to 94 GHz. The quest for even better foreground prediction and subtraction will be an important part of future CMB work as finer details of the temperature pattern are probed with increasing precision. This is likely to be particularly challenging for polarization studies, in which the contamination is larger in proportion to the CMB signal than for the temperature field (Davies & Wilkinson 1998; Sazhin, Sironi & Khovanskaya 2002). It is therefore extremely important to find ways of testing that foreground modelling and subtraction has been carried out accurately, particularly given the evidence that exists already for un-

usual features on the CMB sky revealed by WMAP (e.g. Chiang et al. 2003).

In this paper we show how it is possible to construct an independent probe of foregrounds using measurements of Faraday rotation along lines of sight to extragalactic objects. The layout of the paper is as follows. In the next section we give a brief overview of possible foreground contaminations of CMB maps, and explain how they might be correlated with Faraday rotation measures taken along random lines-of-sight. In Section 3, we carry out a simple and robust test to verify that there is a link between the strength of the sources and their location in the sky. In Section 4 we explain our non-parametric cross-correlation technique. In Section 5, we confirm the link between the RM values and Galactic foregrounds by determining correlations between the RM catalogue and maps of various contributions, including the survey of Haslam et al. (1982) at 408 MHz, synchrotron maps compiled by the WMAP team, as well as dust and free–free maps. We also explore correlations between the RM-values and the temperature field values in the DMR and WMAP data at the location in the sky of the sources to see if there is any evidence of residual contamination in published maps. The conclusions are presented and discussed in Section 6.

2   FARADAY ROTATION AND FOREGROUNDS

Faraday rotation measures (RM) of extragalactic radio sources are direct tracers of the Galactic magnetic field. When plane-polarized radiation propagates through a plasma with a component of the magnetic field parallel to the direction of propagation, the plane of polarisation rotates through an angle ψ given by

$$\psi = \mathcal{R}\lambda^2, \tag{1}$$

where the Faraday rotation measure (RM) is denoted by the symbol $\mathcal{R}$ and measured in $\mathrm{rad\,m^{-2}}$ where

$$\mathcal{R} = \frac{e^3}{2\pi m_e^2 c^4} \int n_e B_\parallel \, ds. \tag{2}$$

Note that $B_\parallel$ is the component of the magnetic field along the line-of-sight direction. The observed RM of extragalactic sources is a linear sum of three components: the intrinsic RM of the source (often small); the value due to the intergalactic medium (usually negligible); and the RM from the interstellar medium of our Galaxy (Broten, MacLeod & Vallée 1988). The latter component is usually assumed to form the main contribution to the integral. If this is true, studies of the distribution and strength of RM values can be used to map the Galactic magnetic field (Vallée & Kronberg 1975). Even if the intrinsic contribution were not small, it could be ignored if the magnetic fields in different radio sources were uncorrelated and therefore simply add noise to any measure of the Galactic field (Frick et al. 2001). In a similar vein, the distributions of RM values have been used to measure local distortions of the magnetic field, such as loops and filaments, and attempts have also been made to determine the strength of intracluster magnetic fields (Kim, Tribble & Kronberg 1991). In what follows we shall use RM values obtained from a catalogue of extragalactic sources

compiled by Broten et al. (1988) (updated in 1991) not to attempt mapping the Galactic B-field but to look for statistical correlations over the whole sky.

There are various ways in which rotation measures of external galaxies could be diagnostic of foreground contamination. The most obvious at first sight is Galactic synchrotron. The *magnitudes* of rotation measures of extragalactic sources trace the Galactic magnetic field strength which, in turn, is correlated with the strength of synchrotron emission resulting from the acceleration of electrons in the Galactic magnetic field. The emission is dependent on both the energy spectrum of the electrons, $N(E)dE$ and the strength of the magnetic field, B. For a power-law distribution of energies of the form

$$N(E)dE = \kappa E^{-\beta} dE, \tag{3}$$

the intensity spectrum of the emitted radiation takes the form

$$I(\nu) \propto \kappa B_\perp^{\alpha+1} \nu^{-\alpha}. \tag{4}$$

The normalisation constant κ is related to the overall number density of electrons, n_e. The intensity spectral index α is related to the index of the energy spectrum β via $\beta = 2\alpha+1$, but α is expected to vary with both position and frequency (Lawson et al. 1987). Radio surveys at frequencies below 2 GHz are dominated by synchrotron emission (Davies & Wilkinson 1998) so these have been used to extrapolate this contribution to the higher frequencies at which CMB instruments operate. However, the only complete-sky survey is that of Haslam et al. (1982) at 408 MHz. Extrapolating from such a low frequency measurement is prone to problems with zero levels and scanning errors. Moreover, it is known that the spectral index α varies by $\Delta\alpha \simeq 0.5$ (Davies, Watson & Gutierrez 1996) which could have a big effect on extrapolations over a large frequency range. If the DMR or WMAP data correlate with RM measurements then this may suggest that synchrotron emission has not been completely removed. Our approach is intended to be complementary to the standard extrapolation.

On the other hand, correlation with Galactic synchrotron is not the only possible diagnostic use of rotation measure data, and may indeed not be the most important. Note that, while the formulae for both rotation measure (2) and synchrotron intensity (4) both depend on B, the former depends on the line-of-sight component and the latter on the perpendicular component. If the magnetic field were disordered on a relatively small scale one might still expect correlations to exist between the two, but it is certainly possible to imagine field configurations that result in large synchrotron emission but no Faraday rotation (and vice-versa). On the other hand, although free–free emission does not rely upon the presence of a B-field, it does depend on the electron density as does the rotation measure. A correlation between RM and free–free emission is therefore possible even if the field configuration leads to a negligible correlation with synchrotron emission in the observer direction. Dust may likewise be indirectly correlated; this correlation may be further complicated if the dust is aligned in some way with the Galactic magnetic field.

It is not obvious *a priori* which of the potential contaminants would correlate best with the measured RM values nor what the meaning of any measured correlation would

be. For example, free–free, synchrotron and dust emission all vary strongly with Galactic latitude. For this reason alone they are expected to correlate with each other. A cross-correlation with RM could therefore, on the face of it, simply be a Galactic latitude effect. On the other hand, such a cross-correlation could instead indicate more complex, smaller-scale spatial association between these foregrounds. We therefore adopt an entirely empirical approach to this question; we return to the issue of Galactic dependence later, in Section 5.

3 THE DISTRIBUTION OF ROTATION MEASURES

Broten et al. (1988) present an all-sky catalogue of rotation measures for 674 extragalactic sources (i.e. galaxies or quasars). It is believed that very large RM values do not reflect the contribution of the Galactic magnetic field, but are instead due to the magnetic field in the source or are perhaps simply unreliable determinations of RM (Ruzmaikin & Sokoloff 1979). The catalogue contained 39 sources where $|\mathcal{R}| > 300 \, \mathrm{rad \, m^{-2}}$; 25 of these are within the Galactic "cut" of the DMR data used in Section 5.2 and 27 are within the Kp2 mask region of the WMAP data used later in Section 5.3. While noting this complication, we decided nevertheless to use the complete sample for this study. The large-valued rotation measures would only be expected to dilute observed correlations if they were entirely intrinsic; we comment on this later, in Section 5.

Our first point of investigation was to look at the distribution of rotation measures across the sky. If we have a set of points on the celestial sphere labelled with some particular characteristic that is independent of the direction on the sphere, any subsample selected using this characteristic should display the same behaviour as the sample as a whole. In particular, any measure of the spatial correlations of the subsample should have the same form as the complete sample (scaled to take into account the smaller sample size). Looking at the clustering characteristics of subsamples of the Faraday rotation measure catalogue can thus indicate whether the RM values for the sources are intrinsic or determined by their spatial positions.

Owing to the small size of the sample available for this analysis, a simple but robust statistic is needed. We chose the conditional distribution of neighbour distances, $\eta(\theta)$, i.e. the probability that there is a neighbour within an angle θ of a given point. This has the advantage of being closely related to the angular two–point correlation function (Peebles 1980). The function $\eta(\theta)$ is straightforwardly calculated by determining the angle between the ith source and each remaining source, and repeating the process for every i. The resulting angles were placed in 100 bins of equal width leading to the distribution of $\eta(\theta)$.

The sample was divided into two sets; those with positive RM values (364 sources) and those with zero or negative values of RM (310 sources). We calculated $\eta(\theta)$ for two subsamples: those with positive rotation measures and those with negative ones, giving $\eta^{+}(\theta)$ and $\eta^{-}(\theta)$ respectively. The two distributions were compared through Monte Carlo (MC) simulations in which the locations of the sources in the sky were maintained, but the RM values were randomly reallo-

cated to the position of the sources. This method ensures the underlying distribution of $\eta(\theta)$ for real and MC samples was the same, but any link between the RM values and spatial position would be severed in the MCs. We extract $\eta^{+}(\theta)$ and $\eta^{-}(\theta)$ from each simulation and constructed an average of distribution over 1000 simulations. Since the distributions are binned it is natural to compare MC distributions with the real data to the average distributions via a χ^2 test, using
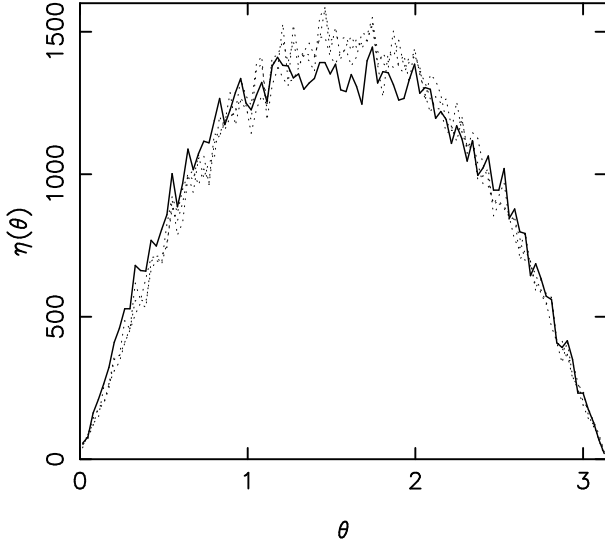
$$\chi^2 = \sum_i \frac{[\eta(\theta_i) - \overline{\eta}(\theta_i)]^2}{\overline{\eta}(\theta_i)}, \tag{5}$$

where $\overline{\eta}(\theta)$ is the average distribution. The larger χ^2 is the less likely the distribution is drawn from the population represented by $\overline{\eta}(\theta)$. The χ^2 statistic obtained from the positive distributions for the MCs and Broten et al. data were ranked (lowest value to highest). The same was done for the statistics obtained from the negative distributions. Thus, the higher the rank of the real data the more likely the rotation measures and positions are correlated.

The distribution $\eta^{-}(\theta)$ calculated from the Broten et al. catalogue and sample Monte Carlo simulations are shown in Fig.1. The distributions of $\eta^{+}(\theta)$ are not shown, but they behave in a similar fashion. The χ^2 values obtained when comparing both subsamples distributions with the average MC distributions were larger than those obtained by all 1000 MC subsamples. Indeed, Fig.1 demonstrates that there are visible differences between the measured distribution and the MC simulations. This demonstrates that the real data show a significant correlation between the sign of rotation measure and the source's spatial location. This fits in with the idea that there is an asymmetry about the meridian about the Galactic centre; a dominance of negative rotation measures at $0 < l < 180^{\circ}$ and positive RM at $180^{\circ} < l < 360^{\circ}$ (Vallée & Kronberg 1975; Han et al. 1997). Furthermore, the average distribution of both subsamples are identical in shape; the peaks and troughs correspond to the same angles. This reinforces the view that both distributions yield similar information. Overall the results support the view that rotation measures trace the Galactic magnetic field at the angular location of the source, which is encouraging for this study.

4 CORRELATING ROTATION MEASURES WITH SKY MAPS

In order to look for correlation between the temperature measurements and the rotation measures at the locations of the sources we use a non-parametric measure of correlation. The temperature (T_i) at the location of source i and the rotation measure $\mathcal{R}_i$ of source i are drawn from unknown probability distributions. However, if the value of each T_i is replaced by the value of its rank among all the other T_i then the resulting list of numbers will be drawn uniformly from integers between 1 and the sample size N (which is 674 for the whole sample). The same procedure is followed for the $\mathcal{R}_i$. We are interested in the magnitude of $\mathcal{R}$ as this is determined by the magnetic field strength; the sign is irrelevant to the intensity of emission at that point. If measurements share the same value then they are assigned the mean of the ranks that they would have had if their values slightly dif-

Figure 1. Distribution of $\eta(\theta)$ with θ for sources with negative rotation measures. The vertical axis is dimensionless and the horizontal axis is θ in radians. The solid line corresponds to the Broten et al. distribution, and the dotted lines show three randomly chosen MC distributions. Note the slight but statistically significant excess at small θ.

fered. In all case the sum of all ranks should be $N(N+1)/2$, where N is the number of sources.

The Spearman rank-order correlation coefficient is constructed as follows. If x_i is the rank of T_i among the other T_i's, and y_i's is the rank of $\mathcal{R}_i$ among the other $\mathcal{R}_i$. Then the Spearman rank-order correlation coefficient, r_s is then given by

$$r_s = \frac{\sum_i (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_i (x_i - \overline{x})^2}\sqrt{\sum_i (y_i - \overline{y})^2}} \qquad (6)$$

A perfect positive correlation is represented by $r_s = 1$, whereas $r_s = -1$ is a perfect negative correlation. This is a better statistic than the usual product-moment correlation coefficient because there is no reason to suppose a linear relation between the two variables. In more general terms it is worth stressing that the non-parametric nature of the Spearman test renders it insensitive to highly skewed distributions. What it measures relates to the ordering of the measurements rather than their actual values so the shapes of marginal distributions of x and y are irrelevant.

In order to establish the significance of a non-zero r_s value obtained from comparing the two sets of measurements, the value was compared to those obtained through MC simulations. The simulations were designed such that the temperature field at a particular point was not linked to the rotation measures. However, the sources could not be simply placed randomly in any location the sky since the real data is clustered which may affect the significance level of any result. There are two ways this could have been simulated to take this into account: the rotation measures of the sources could be shuffled, or the coordinate frame of the source positions could be rotated. The former option was chosen as it is computationally faster. We performed 10,000 MCs and a value of r_s was obtained for each one. This allows us straightforwardly to establish the significance level

of any measurement in terms of the fraction of simulated r_s values exceeded by the real measurement.

5 APPLICATION TO SKY MAPS

5.1 Foreground Maps

The idea of using rotation measures to hunt for Galactic contamination in CMB data hinges on the assumption that foregrounds are related (directly or indirectly) with the Galactic magnetic field. As we explained above in Section 2, this assumption seems theoretically well-founded but not empirically proven. If the two sets of measurements were found to be uncorrelated, it could be that our understanding of the physics of foreground emission needs adjusting or that the belief that the the main contribution to the integral in equation (2) relates to the Galactic magnetic field might be incorrect. In order to verify that there is indeed a correlation between synchrotron emission and rotation measures we looked at a number of "foreground" maps.

To start with, we looked at 8 "pure" synchrotron maps: three of these are derived from the 408 MHz survey of Haslam et al. (1982) and 5 are produced by the WMAP team (Bennett et al. 2003); maps for the latter (and of the data we use later in this section) are available in HEALPix[★] format (Gorski, Hivon & Wandelt 1999) from the NASA archive at http://lambda.gsfc.nasa.gov. The WMAP maps have a resolution parameter of $N_{\text{side}} = 256$ corresponding to $12 \times 256^2 = 786,432$ pixels. The 408 MHz map has a resolution of 51 arcmin and has been reproduced in HEALPix format with $N_{side}=128$. We also looked at this survey smoothed to COBE resolution ($N_{side}=32$). The WMAP team produced their maps using a maximum entropy method (MEM) with the 408 MHz map furnishing a prior spatial distribution; the details of the method are described in Bennett et al. (2003).

The first 8 rows of Table 1 summarise the results from these maps. The method identifies weak but highly significant correlations between the rotation measures and the 8 maps. The WMAP-derived maps yield particularly strong signals, with measured r_s values exceeding those in all 10,000 MCs except for the W-band derived case. These strongly significant correlations add weight to the belief of the WMAP team that they have produced synchrotron-only maps for the 5 frequencies. While the 408 MHz map is at such a low frequency that synchrotron emission completely dominates the sky, the WMAP maps are produced at frequencies where the CMB is significant. Furthermore, these maps have been produced by trying to model and remove other sources of contamination (dust and free–free). The r_s values decrease with frequency across the WMAP range. This is not entirely unexpected, because synchrotron emission (4) decreases with frequency. On the other hand simply boosting or suppressing the amplitude should not influence the non-parametric correlation if the spatial distribution did not change with frequency. This effect may therefore be an artifact of variations in α.

To explore the possible relationship between free–free radiation and RM values we used two maps: one by

Map	r_s	Correlation Probability
Synchrotron		
408 MHz (Unmasked)	0.267	1.00
408 MHz (COBE resolution)	0.277	1.00
408 MHz (Masked)	0.237	1.0o
K derived	0.289	1.00
Ka derived	0.261	1.00
Q derived	0.196	1.00
V derived	0.168	1.00
W derived synchrotron map	0.094	0.99
Free-Free		
Hα (Unmasked)	0.444	1.00
Hα (Masked)	0.496	1.00
WMAP MEM (Unmasked)	0.427	1.00
Dust		
FDS model (Unmasked)	0.405	1.00
FDS model (Masked)	0.429	1.00
WMAP MEM (Unmasked)	0.247	1.00

Table 1. Significance of correlations of Galactic foreground synchrotron, free–free and dust maps described in the text with the Broten et al. Faraday rotation measures.

Map	r_s	Correlation Probability
31.5 GHz channel A	0.039	0.80
31.5 GHz channel B	0.013	0.61
53.0 GHz channel A	-0.074	0.06
53.0 GHz channel B	-0.024	0.29
90.0 GHz channel A	0.062	0.91
90.0 GHz channel B	-0.031	0.26
31.5 GHz (A-B)/2	0.006	0.56
53.0 GHz (A-B)/2	-0.026	0.29
90.0 GHz (A-B)/2	0.059	0.90
31.5 GHz (A+B)/2	0.017	0.64
53.0 GHz (A+B)/2	-0.045	0.17
90.0 GHz (A+B)/2	0.045	0.83
31.5 GHz (A+B)/2 smoothed	0.056	0.89
53.0 GHz (A+B)/2 smoothed	0.002	0.51
90.0 GHz (A+B)/2 smoothed	-0.048	0.15

Table 2. Significance levels of correlations between the DMR maps described in the text and the Broten et al. data.

Finkbeiner (2003), which is a composite all-sky Hα map using data from various surveys, and the other produced by the WMAP team at the K-band frequency using the MEM approach mentioned above. Perhaps surprisingly, the results in Table 1 demonstrate a stronger correlation between these maps and RM than there is in the synchrotron maps, suggesting that the correlation may relate to regions of where the Galactic magnetic field is largely constant or varies parallel to the line-of-sight but in which the electron density fluctuates. Free-free radiation will depend on the square of the electron density so fluctuations in n_e in a region with constant B may produce larger of correlations of RM of free–free emission than synchrotron.

As far as dust is concerned we again use two maps: one is a map constructed by the WMAP team at 94 GHz using models produced by Finkbeiner, Davis & Schlegel (1999) and the other is produced by the WMAP team using MEM at the W-band frequency. Once again the resulting correlations are significantly stronger than those displayed by the synchrotron maps.

One possible explanation of the free–free and dust results is that the cross-correlation with RM we are seeing is basically an effect of Galactic latitude. If all sources (and B) vary with Galactic latitude then they will correlate with each other. This is possible, but one piece of evidence against this explanation is that if we mask out low Galactic latitudes using the Kp2 mask advocated by Hinshaw et al. (2003), the results for synchrotron do not decrease dramatically while those for dust and free–free actually increase slightly.

As a final comment it is worth noting that we repeated the cross-correlation analysis of these maps, excluding the sources in the Broten et al. catalogue that yield very large rotation measures, i.e. $\mathcal{R} > 300$ rad m^{-2}. The resulting values

of r_s were all slightly reduced by a maximum of about 0.04 in r_s. What is interesting about this result is that it shows that these sources probably do contain some information about the Galactic magnetic field because their RM measures are not entirely intrinsic. If these sources only added noise to the cross-correlation, as we suggested above that they might, then the result on the cross-correlation of excluding them would be random rather than systematic.

5.2 COBE DMR Data

The DMR instrument comprised six differential microwave radiometers: two nearly independent channels, labelled A and B, at frequencies 31.5, 53 and 90 GHz. We use data from the region $|b| > 20^{\mathrm{o}}$ with custom cutouts near the Galactic centre (Banday et al. 1997). There are 460 sources from the Broten et al. catalogue in this region to compare with the DMR data. The dipole anisotropy of amplitude ~ 3 mK is largely removed in pre-map-making process (Banday et al. 2003). We chose to look for correlations in 15 maps from the DMR data: the raw data from each of the six radiometers; the (A+B)/2 sum maps and (A-B)/2 difference maps for each frequency; and the (A+B)/2 sum maps after smoothing with a 7° beam up to $l=20$ using the 'smoothing' routine in the HEALPix package. The sum maps should represent the true CMB signal whereas the difference maps should measure the level of instrument noise. The COBE-DMR four year sky maps used for this analysis have a resolution parameter of $N_{\mathrm{side}} = 32$ corresponding to $12 \times 32^2 = 12,288$ pixels in the HEALPix representation.

The results of looking for correlations between the DMR maps and the rotation measures are shown in table 2. The sum maps (corresponding to CMB + contaminants) should show the strongest signs of correlations if there are any to be seen. However, none of the maps show any evidence of correlation with the Broten et al. catalogue. In order to see how stable the probability values are, the simulations were repeated for the 90 GHz (A+B)/2 smoothed map a fur-

ther three times giving probabilities of 0.16, 0.15 and 0.15. This indicates that the probabilities and thus any conclusions drawn from them are valid. Although the main factor may well be the relatively low signal-to-noise in the COBE-DMR data, we can say that our method shows no evidence for any residual foreground component in these maps.

5.3   WMAP 1 yr Sky Maps

The WMAP instrument comprises 10 differencing assemblies (consisting of two radiometers each) measuring over 5 frequencies ($\sim$23, 33, 41, 61 and 94 GHz). The two lowest frequency bands (K and Ka) are primarily Galactic foreground monitors, while the highest three (Q, V and W) are primarily cosmological bands (Hinshaw et al. 2003). For CMB analyses, it is necessary to mask out regions of strong foreground emission. Bennett et al. (2003; B03) provide masks for excluding regions where the contamination level is large. The masks are based on the K-band measurements, where contamination is most severe. The masks, of differing levels of severity, are available from the NASA archive. The severity of the mask is a compromise between eliminating foregrounds and maximising sky area in analyses. We chose to use the Kp2 mask used by Hinshaw et al. (2003) to calculate cross-spectra from the three high frequency data leading to the angular power spectrum. The mask removes 15 % of pixels (including bright sources) leaving 338 faraday sources to compare with the maps. There should be no correlations between the rotation measures and the high frequency maps, once the mask has been applied. If there are correlations, this would question whether the amplitude of the angular power spectrum is cosmological in origin.

The WMAP team have also released an internal linear combination (ILC) map that combined the five band maps in such a way to maintain unity response to the CMB whilst minimising foreground contamination. The construction of this map is described in detail in Bennett et al. (2003). To further improve the result, the inner Galactic plane is divided into 11 separate regions and weights determined separately. This takes account of the spatial variations in the foreground properties. Thus, the final combined map does not rely on models of foreground emission and therefore any systematic or calibration errors of other experiments do not enter the problem. The final map covers the full-sky and should represent only the CMB signal.

Following the release of the WMAP 1 yr data Tegmark, de Oliveira-Costa & Hamilton (2003; TOH) have produced a cleaned CMB map. They argued that their version contained less contamination outside the Galactic plane compared with the internal linear combination map produced by the WMAP team. The five band maps are combined with weights depending both on angular scale and on the distance from the Galactic plane. The angular scale dependence allows for the way foregrounds are important on large scales whereas detector noise becomes important on smaller scales. TOH also produced a Wiener filtered map of the CMB that minimises rms errors in the CMB. Features with a high signal-to-noise are left unaffected, whereas statistically less significant parts are suppressed. While their cleaned map contains residual Galactic fluctuations on very small angular scales probed only by the W band, these fluctuations vanish in the filtered map.

Map	r_s	Correlation Probability
K	0.292	1.00
Ka	0.189	1.00
Q	0.149	1.00
V	0.126	1.00
W	0.063	0.95
K (with mask)	0.296	1.00
Ka (with mask)	0.099	0.97
Q (with mask)	0.041	0.77
V (with mask)	0.049	0.81
W (with mask)	0.043	0.78
Internal linear combination map	0.021	0.71
TOH cleaned map	0.059	0.93
TOH Wiener map	0.068	0.96

Table 3. Significance levels of cross-correlations derived from various maps originating from WMAP with the Broten et al. catalogue.

In all, 13 maps derived from the WMAP data were used to seek correlation with the Broten et al. catalogue: the five band maps, the five band maps with the Kp2 region removed, the internal linear combination map, and the cleaned and Wiener maps of TOH.

The results of looking for correlations between the WMAP derived data and the rotation measures are shown in Table 3. The uncut frequency maps are all significantly correlated with the rotation measures, except the highest frequency map (W band). This confirms the expected contamination of the data across these frequencies. The strength of correlation decreases with increasing frequency, which may be understood by looking at the contribution of all the foregrounds to the total sky signal across these bands. Combining the contributions of synchrotron, free–free and dust, with sychrotron given a weighting of 0.5 due to its weaker correlation, would reproduce this trend. Once the Kp2 mask has been applied, the correlations vanish for all but the K and Ka band map. The reason that a correlation is still found with the K band data is probably that the cut only excludes the very strongest signals so both bands are still heavily contaminated. This is probably not a problem for CMB analysis because these will be projected out in the likelihood analysis. Moreover, the results from the masked maps indicate that calculating the angular power spectrum from the 3 high frequency bands with the Kp2 cut is valid. The contamination from Galactic foregrounds correlated with Faraday rotation on these studies is therefore probably small. Finally, no correlations are found between the rotation measures and two of the CMB-only maps, suggesting that the levels of contamination are indeed low, as the authors claim. Nevertheless, the result from the Wiener filtered map of TOH suggests a correlation at 95% confidence level.

Finally, we mention that on the NASA archive there are maps from each high frequency assembly that are supposedly clean of foreground contaminants outside the Kp2 cut region. The results in Table 4 show there are indeed no

Map	r_s	Correlation Probability
q1 cleaned	0.005	0.53
q2 cleaned	-0.075	0.08
v1 cleaned	-0.008	0.43
v2 cleaned	0.023	0.66
w1 cleaned	-0.048	0.19
w2 cleaned	0.009	0.56
w3 cleaned	-0.047	0.19
w4 cleaned	0.018	0.63

Table 4. Cross-correlations in the "clean maps" from each high frequency assembly

significant residual correlations in these data when the Kp2 mask is applied.

6   DISCUSSION AND CONCLUSIONS

The aim of the paper was to look for traces of Galactic contamination in the COBE-DMR 4 yr data and WMAP 1 yr data using correlations between the maps and the rotation measures of Broten et al. (1988).

We first studied the relationship between the spatial position and the RM values of the sources in the catalogue by looking at the angular correlations of subsets drawn from it. The results clearly indicate a correlation, and confirm the basic view expressed by other authors on properties of the Galactic magnetic field. We then investigated the relationship between the rotation measure of a source and the synchrotron emission at the location of the source. Synchrotron maps derived from the 408 MHz survey of Haslam et al. (1982) and produced by the WMAP team were found to be correlated with RM values, showing that these do indeed provide some sort of probe of the Galactic magnetic field.

We found *stronger* positive correlations of RM values with both dust and free–free maps than for synchrotron. This is consistent with indirect association of the different sources, but is probably not simply a Galactic latitude effect because the correlation persists even when a Galactic cut is applied. Exactly how this correlation arises is an issue for further study, but it may provide an insight into the possible role of spinning dust (Draine & Lazarian 1998) in Galactic foregrounds (de Oliveira-Costa et al. 1999) as this may align with the local magnetic field. Clearly much more detailed modelling is needed to understand these correlations theoretically, but the empirical approach we have adopted still provides a useful consistency check on foreground analysis even if the origin of the correlation is not well understood.

The correlations between the temperature strength of the DMR and WMAP data at the location of sources and the RM values were then studied using the Spearman rank-order correlation coefficient. All 15 maps compiled from the temperature field measured by the DMR instrument were found to be uncorrelated with the rotation measurements. Correlations were found with the uncut WMAP frequency maps and the cut K and Ka band maps. However, the maps used by the WMAP team to extract cosmological information were found to be uncorrelated. Furthermore, two foreground-subtracted CMB-only maps were found to be uncorrelated with the rotation measure catalogue. The results of this analysis provide no evidence of residual foregrounds in the COBE-DMR maps or the WMAP ILC map, but do yield a positive correlation for the Tegmark Wiener-filtered map. Owing to the small size of our RM sample we used these results are only suggestive, but they do demonstrate the virtue of looking for independent probes of Galactic foreground contamination. Much larger compilations of RM values would be needed to make more definite statements about contamination in temperature maps. It is also very likely that much could be learnt about polarized foregrounds using a similar approach.

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